

# Structure of meson-baryon interaction vertices

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We present a microscopic derivation of the form factors of strong-interaction  $\pi NN$  and  $\pi N\Delta$  vertices within a relativistic constituent quark model. The results are compared with form factors from phenomenological meson-baryon models and recent lattice QCD calculations. We give an analytical representation of the vertex form factors suitable for applications in further studies of hadron reactions.

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Understanding the meson-baryon strong-interaction vertices has been a hard and long-standing problem. Attempts to derive a microscopic explanation, desirably on the grounds of QCD, have not yet led to conclusive results. The problem is of considerable importance not only in particle but also in nuclear physics. Practically all realistic meson-exchange  $NN$  potentials,  $3N$  forces, and  $\pi N$  dynamical models rely on certain inputs for strong form factors. Mostly they have been based on phenomenological arguments and one has usually employed monopole or dipole parametrizations with cut-off parameters fitted to experiment. Different parametrizations have big influences, e.g., on meson-baryon dynamical models (see Refs. [1, 2, 3, 4, 5, 6]), on  $NN$  potentials often used in present-day nuclear calculations (e.g., the Nijmegen [7, 8, 9], Bonn [10, 11, 12], and Argonne [13, 14] potentials), and on  $3N$  forces, see, e.g., refs. [15, 16]. Consequently, a microscopic derivation of the meson-baryon interaction vertices constitutes an important problem and it has long and often been asked for (see, e.g., Refs. [17, 18]).

The uncertainty about the meson-baryon strong-interaction vertices has even been increased by the recent advent of lattice QCD calculations [19, 20]. These works have led to results different among each other and partly distinct from earlier lattice QCD calculations by Liu *et al.* [21, 22]. Lattice QCD results are furthermore at variance with form factors adopted so far in relativistic models of meson-baryon dynamics [4, 5, 6].

Here, we perform a microscopic derivation of the strong meson-baryon form factors on the basis of a relativistic constituent quark model (RCQM). It is free of any phenomenological input (fit parameters), and the form-factor dependence on the relativistic four-momentum transfer  $Q^2$  is directly predicted from the RCQM, which has already been successful in reproducing the invariant mass spectrum of baryons [23, 24] and the electroweak structure of the nucleons and other baryon ground states [25, 26, 27, 28, 29]. The same RCQM has recently been employed in a covariant study of the mesonic decays of baryon resonances [30, 31, 32] leading to results for partial decay widths qualitatively rather different from previous nonrelativistic or relativized studies. The systematics found in the relativistic decay widths for all the  $\pi$ ,  $\eta$ , and  $K$  decay modes has subsequently also led to a partly new classification of baryon resonances into flavor multiplets [33, 34].

In this paper we consider the  $\pi NN$  and  $\pi N\Delta$  form factors according to the process depicted in Fig. 1(a) and described by the matrix elements of the hadronic interaction Lagrangian

$$F_{i \rightarrow f} = (2\pi)^4 \langle f | \mathcal{L}_I(0) | i \rangle, \quad (1)$$

where  $f$  denotes the meson-emitting baryon and  $i$  the final meson-nucleon state. The interaction Lagrangian densities  $\mathcal{L}_I$  are given by

$$\mathcal{L}_I^N = -\frac{f_{\pi NN}}{m_\pi} \bar{\Psi}(x) \gamma_5 \gamma^\mu T \Psi(x) \partial_\mu \Phi(x) \quad (2)$$

$$\mathcal{L}_I^\Delta = -\frac{f_{\pi N\Delta}}{m_\pi} \bar{\Psi}(x) T \Psi^\mu(x) \partial_\mu \Phi(x) + h.c. \quad (3)$$

Herein,  $\Psi$  is the nucleon Dirac field,  $\Psi^\mu$  the  $\Delta$  Rarita-Schwinger field,  $\Phi$  the meson field, and  $T$  represents the transition operator for the meson-emission process;  $f_{\pi NN}$  and  $f_{\pi N\Delta}$  are the  $\pi NN$  and  $\pi N\Delta$  coupling constants, respectively. We identify this process with the matrix elements of the (reduced) transition operator  $\hat{D}_{\text{rd}}^\pi$  for the same process sandwiched between eigenstates of the invariant mass operator of the RCQM [30]

$$F_{i \rightarrow f}^{\text{RCQM}} = \langle V', M', J', \Sigma' | \hat{D}_{\text{rd}}^\pi | V, M, J, \Sigma \rangle, \quad (4)$$

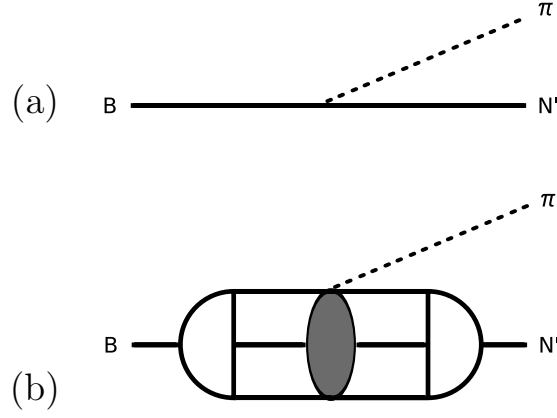


FIG. 1: Graphical representation of the meson-baryon vertex (a) and the corresponding amplitude in the RCQM (b).

graphically shown in Fig. 1(b). The baryon eigenstates in Eq. (4) are characterized by the four-velocity  $V$ , the invariant-mass eigenvalue  $M$ , and the intrinsic spin  $J$  with z-component  $\Sigma$ . We calculate the transition amplitude (4) in the point form of Poincaré-invariant quantum mechanics and take the transition operator according to the spectator model [35], i.e.

$$\begin{aligned} \langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{D}_{\text{rd}}^\pi | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle = \\ 3\mathcal{N} \frac{ig_{qqm}}{2m_1 (2\pi)^{\frac{3}{2}}} \bar{u}(p'_1, \sigma'_1) \gamma_5 \gamma^\mu \lambda_m u(p_1, \sigma_1) \tilde{Q}_\mu \\ 2p_{20} \delta(\vec{p}_2 - \vec{p}'_2) 2p_{30} \delta(\vec{p}_3 - \vec{p}'_3) \delta_{\sigma_2 \sigma'_2} \delta_{\sigma_3 \sigma'_3}, \end{aligned} \quad (5)$$

where the quark-meson coupling constant  $g_{qqm}$  is fixed to the same value of  $\frac{g_{qqm}^2}{4\pi} = 0.67$  as used in the Goldstone-boson-exchange (GBE) RCQM [23, 24]. The off-shell extrapolation of the transition amplitude is made by keeping all hadrons and quarks on their respective mass shells. Obviously it implies energy non-conservation in the transition process. By virtue of the pseudovector-pseudoscalar equivalence the above construction also guarantees that the pseudovector and pseudoscalar quark-meson couplings lead to the same transition amplitude.

As a function of the invariant four-momentum transfer squared in the space-like region,  $Q^2 = -q^2 > 0$ , the strong  $\pi NN$  form factor in the rest-frame of the meson-emitting baryon is given by

$$G_{\pi NN}(Q^2) = \frac{1}{f_{\pi NN}} \frac{m_\pi \sqrt{2\pi}}{\sqrt{2M_N}} \frac{\sqrt{E'_N + M'_N}}{E'_N + M'_N + \omega} \frac{F_{i \rightarrow f}^{\text{RCQM}}}{Q_z}, \quad (6)$$

where the momentum transfer is taken into the  $z$ -direction. Similarly, the  $\pi N\Delta$  form factor reads

$$G_{\pi N\Delta}(Q^2) = -\frac{1}{f_{\pi N\Delta}} \frac{3\sqrt{2\pi}}{2} \frac{m_\pi}{\sqrt{E'_N + M'_N} \sqrt{2M_\Delta}} \frac{F_{i \rightarrow f}^{\text{RCQM}}}{Q_z}. \quad (7)$$

Thereby we get the predictions of the RCQM for the  $\pi NN$  and  $\pi N\Delta$  coupling constants as well as the  $Q^2$  dependences of the vertex form factors.

The  $Q^2$  dependence of the  $\pi NN$  strong form factor  $G_{\pi NN}$  as predicted by our RCQM is shown in Fig. 2. There a comparison is made to parametrizations from two dynamical meson-baryon models as well as results from lattice QCD calculations. Our results compare best with the  $\pi NN$  form factor of Sato-Lee [4], which represents a bare form factor, i.e. without hadron dressing. For comparison we also give the dressed form factor from another dynamical meson-baryon model, namely the one of Polinder-Rijken [5, 6]. It exhibits a remarkably slower fall off at small  $Q^2$ . The same is true for the lattice QCD results by Liu *et al.* [21, 22] as well as the most recent ones by Erkol *et al.* [20]. The latter essentially agree with each other, where they both have made an extrapolation of their lattice data to the physical pion mass or the chiral limit, respectively. The slowest fall off is shown by two types of lattice QCD calculations of Alexandrou *et al.* [19], namely the one with quenched Wilson fermions and a pion mass of 0.411 GeV (denoted as set A in Fig. 2) and the one with dynamical Wilson fermions and a pion mass of 0.384 GeV (denoted as set B in Fig. 2); both are normalized using the coupling constant from their linear fit.

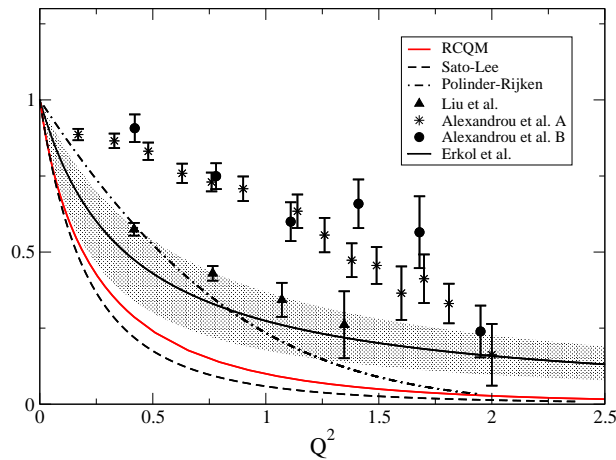


FIG. 2: Prediction of the strong form factor  $G_{\pi NN}$ , normalized to 1 at  $Q^2 = 0$ , by the RCQM (solid/red line) in comparison to parametrizations from the dynamical meson-baryon models of Sato-Lee [4] and Polinder-Rijken [5, 6] as well as results from three lattice QCD calculations [19, 20, 21, 22] (cf. the legend); the shaded area around the result by Erkol *et al.* gives their theoretical error band. See also the explanations in the text.

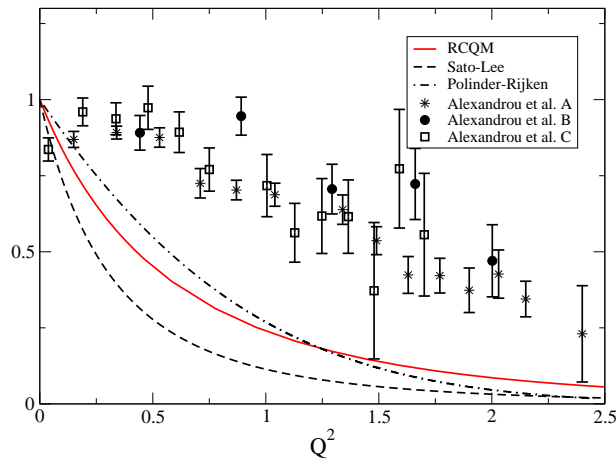


FIG. 3: Same as Fig. 2 but for the strong form factor  $G_{\pi N\Delta}$ .

The analogous predictions for the strong form factor  $G_{\pi N\Delta}$  are given in Fig. 3. In this case there exist lattice QCD calculations only by Alexandrou *et al.* [19]. In addition to the lattice data sets A and B, as in Fig. 2, we have added a further set denoted as C, which corresponds to a calculation with a hybrid action and a pion mass of 0.353 GeV. Again, the lattice QCD results by Alexandrou *et al.* are much above our predictions, while our results now compare reasonably well with the parametrizations in both dynamical models by Sato-Lee [4] as well as Polinder-Rijken [5, 6]. We recall that the form factor of the first corresponds to undressed hadrons whereas the one of the latter is dressed.

It is interesting to observe that the  $Q^2$  dependence of both the  $G_{\pi NN}$  and  $G_{\pi N\Delta}$  form factors resulting directly and in a parameter-free manner from the RCQM qualitatively agrees with the parametrizations of the meson-baryon vertices in the Sato-Lee model [4]. From their work the dressing effect is only visible in the  $\pi N\Delta$  case, where with increasing  $Q^2$  the dressed form factor shows a slightly faster fall off than the bare form factor. There is no direct information on the behavior of undressed form factors in the works by Polinder-Rijken [5, 6]. *A-priori* there is no explicit dressing present in the RCQM yet. In principle, one should start out with bare hadron masses in the construction of a constituent quark model. Then, meson-cloud effects should be included explicitly. This is feasible now and remains as a challenge for future constructions of RCQMs that aim to include the coupling to mesonic channels. Thereby one could finally determine the dressing effects unambiguously in the masses as well as in the form factors.

On the other hand, the strong form factors from the lattice calculations show a (much) slower fall off with increasing

TABLE I: Coupling constants and cut-off parameters of vertex form factors. The results of the RCQM are represented according to the representation (8) and are compared with the phenomenological models by Sato-Lee [4] (SL) as well as Polinder-Rijken [5, 6] (PR). For the lattice QCD calculations by Liu *et al.* [21, 22] (LIU), Erkol *et al.* [20] (ERK) and set A of Alexandrou *et al.* [19] (ALX) the monopole fit of Eq. (9) is applied.

	RCQM	SL	PR		LIU	ERK	ALX
$\frac{f_N^2}{4\pi}$	0.0691	0.08	0.075		0.0649	0.0481	0.0412
$N \Lambda_1$	0.451	0.453	0.940	$\Lambda$	0.747	0.614	1.65
$\Lambda_2$	0.931	0.641	1.102		-	-	-
$\frac{f_\Delta^2}{4\pi}$	0.188	0.334	0.478				
$\Delta \Lambda_1$	0.594	0.458	0.853				
$\Lambda_2$	0.998	0.648	1.014				

$Q^2$ . However, even for the smaller differences between our results (as well as the bare form factors of Sato-Lee) and the data sets by Liu *et al.* and Erkol *et al.* it appears questionable that this would turn out merely as a dressing effect. Regarding all of the lattice data by Alexandrou *et al.* one has also to keep in mind that they correspond to larger pion masses with no extrapolations applied. Thus it remains as an open question if this is responsible for their rather weak  $Q^2$  dependences. In any case calculations/extrapolations towards smaller pion masses would be desirable.

The vertex form factors constitute an important input into all kind of dynamical hadron models. Therefore we present the RCQM predictions shown above in analytical forms suitable for further use. In particular, we adopt a form intermediate between the usual monopole and dipole parametrizations

$$G(\vec{q}^2) = \frac{1}{1 + \left(\frac{\vec{q}}{\Lambda_1}\right)^2 + \left(\frac{\vec{q}}{\Lambda_2}\right)^4}. \quad (8)$$

This particular parametrization of the form factors depends on the three-momentum transfer  $\vec{q}^2$  rather than the four-momentum transfer  $Q^2 = -q^2$ . It also provides enough flexibility to represent our results above, while we could not obtain good fits with an ansatz of either the standard monopole, dipole, or exponential type. The monopole and dipole form factors are in fact contained in our parametrization as limiting cases; Eq. (8) reduces to the monopole form factor for  $\Lambda_2 \rightarrow \infty$  and to the dipole form factor for  $\Lambda_1 = \Lambda_2/\sqrt{2}$ .

The parametrization (8) also allows to compare the form factor results on an equal footing in terms of coupling constants and cut-off parameters. The corresponding values are given in Table I. For the RCQM the size of the coupling constant  $\frac{f_N^2}{4\pi}$  (defined at  $\vec{q}^2 = 0$ ) for  $\pi NN$  compares well with the phenomenological value of approximately 0.075 [36] as is the case for the Sato-Lee and Polinder-Rijken models. The same is still true for the vertex form factors by Liu *et al.*, whereas the more recent lattice QCD calculations by Erkol *et al.* as well as Alexandrou *et al.* yield much too small coupling constants. For the  $\pi N\Delta$  coupling constant  $\frac{f_\Delta^2}{4\pi}$  we obtain a smaller coupling constant than Sato-Lee and Polinder-Rijken. Due to the lack of firm experimental evidence it remains as an open question, which is the most adequate value. In this context, it is noteworthy that Polinder-Rijken found the bare coupling constant to be  $\frac{f_\Delta^2}{4\pi} = 0.167$ , i.e. a much smaller value than the dressed one but closer to our RCQM prediction.

For the  $\vec{q}^2$  dependence of the vertex form factors the best fits are obtained with the cut-off parameters in Table I. We recall that the original results of Sato-Lee and Polinder-Rijken are given as dipole and exponential form factors, respectively. For the lattice QCD calculations by Liu *et al.* as well as Erkol *et al.* there exist monopole form-factor fits in terms of  $Q^2$

$$G(Q^2) = \frac{1}{1 + \left(\frac{Q}{\Lambda}\right)^2}, \quad (9)$$

and the data from the linear fit by Alexandrou *et al.* can also be cast into the same type of monopole representation. As is seen from Table I the cut-off parameter  $\Lambda$  relating to the form-factor results of Alexandrou *et al.* is remarkably high.

In summary, we have presented a parameter-free microscopic description of the strong  $\pi NN$  and  $\pi N\Delta$  vertex form factors with a fully relativistic constituent quark model. The  $Q^2$  dependences of the RCQM form factors are

qualitatively similar to the ones parametrized along the phenomenological dynamical meson-baryon model by Sato-Lee with bare hadrons. The RCQM predictions require a parametrization intermediate between a monopole and dipole form. In particular, this also suggests that there is no preference for a cut-off of exponential type, which has sometimes been claimed to be suggested from microscopic quark-model considerations. In addition, our study reveals that the structure of the  $\pi N\Delta$  vertex is sizably different from the  $\pi NN$  one, with cut-off parameters of up to 25% larger. This is at variance with form factor parametrizations often used in phenomenological models, where the  $\pi NN$  and  $\pi N\Delta$  cut-offs are assumed of similar size [4, 5, 6] or even decreasing in the transition from  $\pi NN$  to  $\pi N\Delta$ .

The lattice results on the vertex form factors are qualitatively distinct from the RCQM predictions and they also differ considerably among each other. It remains to be shown if the slower fall-off of the lattice QCD form factors with increasing  $Q^2$  is just a dressing effect. In this regard it appears most important to have on the one hand more lattice data and on the other hand (relativistic) quark-model studies that can quantitatively pin down the contributions of hadron dressing.

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